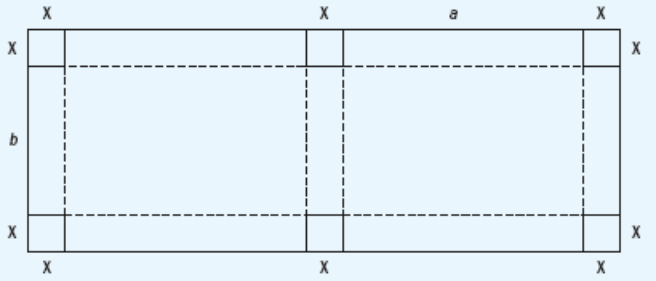
Chapter 4 / **Example 34**

Optimization problems

A piece of cardboard, measuring 100 cm by 200 cm, is to be made into a box by cutting out small squares, each with side length x, as shown in the diagram. a is the length between the squares on the longer side of the cardboard, and b is the length between the squares on the shorter side of the cardboard.

i Find expressions for a and b in terms of x, and state the constraints on the lengths of x, a, and b.

ii Find the value of x (in cm^3) which maximizes the volume of the box, and find the maximum volume of the box. Check your answers graphically.

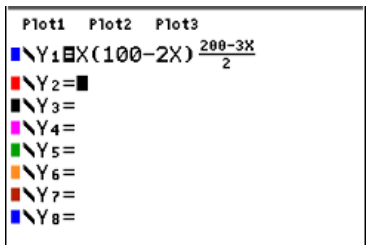


The problem requires the maximization of the function

$V = x(100-2x)\left(\frac{200-3x}{2}\right),\ x < 50$

Press [f1] [y=] to display the equation entry screen.

Type $x(100-2x)\left(\frac{200-3x}{2}\right)$ and press [enter] to enter the equation as Y_1.

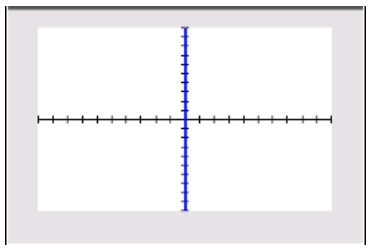


Press [f5] [graph] to display the graph screen

The GDC now displays the function:

$Y_1 = x(100-2x)\left(\frac{200-3x}{2}\right)$ with the default axes.

The default axes are $-10 \le x \le 10$ and $-10 \le y \le 10$.

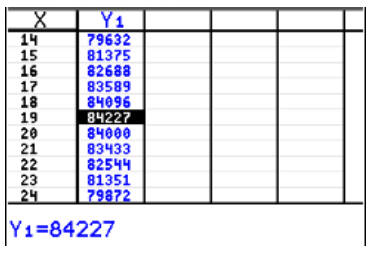


To get a better idea of the best window to view the graph in, it is helpful to use a table of values.

Press [2nd] [f5] [table].

A table of values is displayed.

You can scroll through the table using [▲] and [▼].



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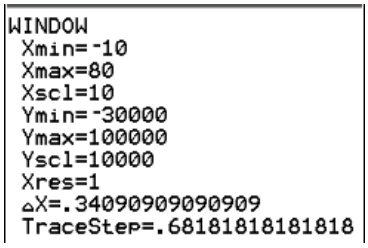
Use this information to choose suitable window settings to display the graph.

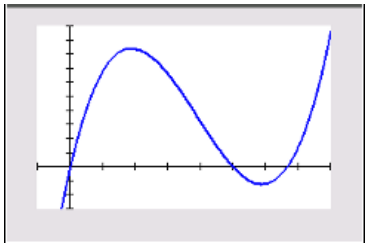
Press [f2] [window] [format]

Set the axes to show $-10 \leq x \leq 80$ with a scale of 10 and $-30\,000 \leq y \leq 100\,000$ with a scale of 10 000.

Press [f5] [graph] when you have finished.



The GDC displays the graph in a suitable window.

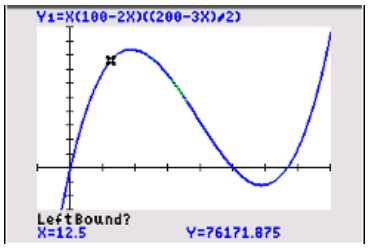


To find the maximum press [2nd] [f4] [calc] 4:maximum.

You will need to give the left and right bounds of the region that includes the maximum.

The GDC shows a point on the curve and asks you to set the left bound. Move the point using [▶] [◀] and choose a position to the left of the turning point.

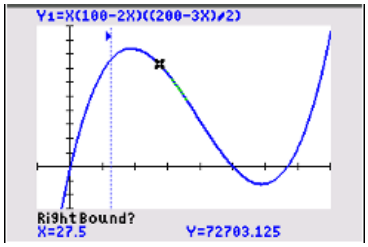
Press [enter].



The GDC shows a line where you have set the left bound and a point on the curve.

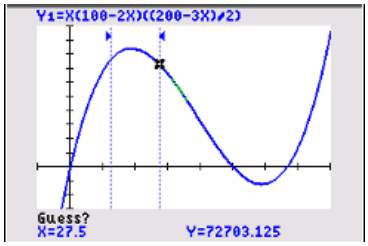
Move the point using [▶] [◀] and choose a position to the right of the turning point.

When the region contains the turning point, Press [enter].



The GDC requires an initial guess for the position of the turning point. Choose the default position.

Press [enter].



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The GDC displays the local maximum point at $(18.9, 84\,200)$.

There is also a minimum point which occurs when $x > 50$ which can be rejected.

So the maximum volume is 84 200 cm^3 when x = 18.9 cm.